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Analysis of Weak Neutral Currents Using Light-Cone Algebra

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Abstract

Using the algebra of light-cone commutators and making some simplifying assumptions, we have attempted to determine the isospin structure of a two-parameter weak hadronic neutral current. We have also computed in the light-cone model, the y distributions for inclusive neutrino and antineutrino neutral current deep-inelastic interactions. Contrary to the quark–parton model, we find that both distributions have the same value.

1. *Introduction*

One of the important discoveries in the current series of high-energy neutrino experiments is the weak neutral current. Previous experiments with high-energy neutrinos showed that interactions of neutrinos with matter always gave rise to a charged lepton in the final state.

Recently, however, neutrino experiments at CERN and Fermilab (Hasert et al., 1973, 1974; Benvenuti et al., 1974a, b; Barish et al., 1974a, b, 1975; Aubert et al., 1974a, b; Cline et al., 1974) showed that besides the well-established charged current processes, one also finds neutral current channels in which an incident neutrino does not change into a charged lepton, but retains its identity.

This interesting experimental observation called immediately for some theoretical analysis and understanding. Among the problems to be understood was that of the space-time and internal symmetry structure of the neutral current, and how this structure could be reconciled with the observed branching ratios of charged and neutral current channels.

This problem has been tackled by several authors (Pais and Treiman, 1974; Rajasekaran and Sarma, 1974; Ecker and Pietschmann, 1975; Ecker 1975; Pakvasa and Rajasekaran, 1975; Cordes, et al., 1975; Wolfenstein, 1974; Paschos, 1975; Pietschmann, 1975) from various points of view and with varying degrees of approximation and success. Basically, two steps are involved in all such calculations. One has first to adopt a suitable parametrization of the neutral current and then calculates, using some model, the deep-inelastic scaling functions. It turns out that the data presently available, or likely to be available in the near future, are not sufficient to allow one the luxury of a fine-structure parametrization of the neutral current. To study the internal symmetry structure of the neutral current, one usually adopts a two-parameter structure. Even then, further simplifying assumptions seem to be called for, in order to determine the two parameters.

In this paper, we have attempted to determine the isopin structure of the neutral current, using the method of light-cone commutators. An earlier application of this method to the same problem is that of Razmi (Razmi, 1974).

2. *Parametrization of Weak Neutal Current*

Considering the weak neutral current as made up of only vector and axial-vector parts, and taking each piece as consisting of isovector and isoscalar parts, one can write the current generally as follows:

$$J_\mu{}^Z = V_\mu^{3,N} + A_\mu^{3,N} + V_\mu^{0,N} + A_\mu^{0,N}$$

The next assumption one could make is that the isotriplet pieces are members of the usual $SU(2)$ currents, while all the four currents are members of an $SU(3)$ set of currents. This requires putting

$$V_\mu^{3,N} \sim J_\mu{}^3, \qquad V_\mu^{0,N} \sim J_\mu{}^8$$
$$A_\mu^{3,N} \sim J_{5\mu}^3, \qquad A_\mu^{0,N} \sim J_{5\mu}^8$$

Starting from these assumptions one can now consider the various ways in which these pieces may combine to give the physical weak neutral current. Some possibilities suggested in the literature (Pais and Treiman, 1974; Beg and Zee, 1973; Sakurai, 1974; Pakvasa and Tuan, 1974; Gourdin, 1975) are summarized here in our Table I. The quantities $\alpha, \beta, \lambda, \rho, \gamma, k, \delta, \sigma, \eta_v, \eta_A$, $d, \ldots$ are unknown coefficients to be determined. For model I, for example,

$$J_\mu{}^Z = \alpha J_\mu{}^3 + \beta J_{5\mu}^3$$

while for model II

$$J_\mu{}^Z = \lambda(J_\mu{}^3 - J_{5\mu}^3) + \rho J_\mu^{\text{em}}$$

3. *The Scaling Functions*

For the process $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + X$, the double differential cross section in the scaling limit has the form

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G^2ME}{\pi}\left[(1-y)F_{2(x)}^{\nu(\bar{\nu})} + xy^2F_{1(x)}^{\nu(\bar{\nu})} \mp x\left(y - \frac{y^2}{2}\right)F_{3(x)}^{\nu(\bar{\nu})}\right] \tag{3.1}$$

TABLE I

$J_\mu{}^Z$	$J_\mu{}^3$	$J_{5\mu}^3$	$J_\mu{}^3 - J_{5\mu}^3$	$J_\mu{}^3 + J_{5\mu}^3$	$J_\mu{}^8$	$J_{5\mu}^8$	$J_\mu^{\mathrm{em}} = J_\mu{}^3 + 3^{-1/2} J_\mu{}^8$	$J_\mu{}^B$	$J_{5\mu}^B$
Model I	α	β	—	—	—	—	—	—	—
Model II	—	—	λ	—	—	—	ρ	—	—
Model III	α	—	—	—	β	—	—	—	—
Model IV	—	—	—	—	β	γ	—	—	—
Model V	—	k	—	—	—	k	ρ	—	—
Model VI	—	γ	—	—	δ	—	—	—	—
Model VII	—	—	—	—	—	—	—	η_v	η_A
Model VIII	—	—	λ	σ	—	—	ρ	—	—
Model IX	—	—	λ	σ	—	—	ρ	d	d
Model X	α	β	—	—	γ	δ	—	—	—

where

$$x = \frac{Q^2}{2M\nu}, \qquad Q^2 = -q^2 = 4EE' \sin^2\left(\frac{\theta}{2}\right)$$

q is the momentum transfer; $y = \nu/E = (E - E')/E$ = inelasticity parameter; M is the nucleon mass; G is the usual Fermi weak coupling constant; while E, E', and θ are, respectively, the incident neutrino energy, the final neutrino energy, and the neutrino scattering angle, in the laboratory system. In order to calculate the scaling functions $F_{i(x)}^{\nu(\bar{\nu})}$, $i = 1, 2, 3$, we consider the hadronic tensor

$$W_{\mu\nu} = \sum_X \bar{\sum_{\text{spins}}} \int \frac{d^4x}{4\pi} e^{iqx} \langle p | [J_\mu^z(x), J_\mu^z(0)] | p \rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(\nu, q^2) + \frac{1}{M^2}\left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)$$

$$\times \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) W_2(\nu, q^2) - \frac{i\epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}{2M^2} W_3(\nu, q^2)$$

Choosing any one of the parametrizations of J_μ^z listed in Table I, we can evaluate our commutator on the light cone. We consider the specific case of model II and obtain

$$W_{\mu\nu} = \frac{1}{4|p \cdot q|} [p_\mu (q + xp)_\nu + p_\nu (q + xp)_\mu - p \cdot (q + xp) g_{\mu\nu}]$$

$$\times \left[(\tfrac{2}{3})(\tfrac{2}{3})^{1/2} A^0(x)(2\rho^2 + 3\lambda^2 + 3\lambda\rho) + (\tfrac{2}{3})^{1/2} A^3(x)(\rho^2 + \lambda\rho)\right.$$

$$\left. + \frac{2}{3(3)^{1/2}} A^8(x)(3\lambda^2 + 3\lambda\rho + \rho^2)\right] + \frac{i\epsilon_{\mu\nu\rho\sigma} p^\delta q^\sigma}{4|p \cdot q|}$$

$$\times \left\{-2\left[(\tfrac{2}{3})^{1/2} \lambda\rho + \lambda^2\right] S^0(x) - \frac{2}{3}\lambda\rho S^3(x) - \frac{2}{\sqrt{3}}(\lambda^2 + \lambda\rho) S^8(x)\right\}$$

where $S^k(x)$ and $A^k(x)$ are certain functions to be evaluated (Murtaza, 1974).

By comparing now with the form factor parametrization of $W_{\mu\nu}$ in the scaling limit, we obtain the following expressions for the scaling functions:

$$F_{1(x)}^{\nu(\bar{\nu})} = \tfrac{1}{4}\left\{A^0(x)[\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2 + 3\lambda^2 + 3\lambda\rho)] + A^3(x)[\tfrac{2}{3}(\rho^2 + \lambda\rho)]\right.$$

$$\left. + A^8(x)\left[\frac{2}{3(3)^{1/2}}(3\lambda^2 + 3\lambda\rho + \rho^2)\right]\right\}$$

$$F_{2(x)}^{\nu(\bar{\nu})} = 2xF_{1(x)}^{\nu(\bar{\nu})} \qquad \text{from the Callan-Gross relation}$$

$$F_{3(x)}^{\nu(\bar{\nu})} = -\tfrac{1}{2}[-2(\tfrac{2}{3})^{1/2}(\lambda^2 + \lambda\rho) S^0(x) - \tfrac{2}{3}\lambda\rho S^3(x) - 2/(3)^{1/2}(\lambda^2 + \lambda\rho) S^8(x)]$$

4. *y Distribution*

Substituting into equation (3.1) we then get that

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy} = \frac{G^2ME}{\pi}\left\{(1-y)\frac{x}{2}\left[\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)A^0(x)\right.\right.$$

$$+A^3(x)\cdot\tfrac{2}{3}(\rho^2+\lambda\rho)+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)$$

$$\left.\left.+\frac{xy^2}{4}[A^0(x)\cdot\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)\right]\right\}$$

$$+A^3(x)[\tfrac{2}{3}(\rho^2+\lambda\rho)]+A^8(x)\left[\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$\mp\frac{x}{2}\left(y-\frac{y^2}{2}\right)[2(\tfrac{2}{3})^{1/2}(\lambda^2+\lambda\rho)S^0(x)+\tfrac{2}{3}\lambda\rho S^3(x)$$

$$\left.+\frac{2}{(3)^{1/2}}(\lambda^2+\lambda\rho)S^8(x)]\right\}$$

For the specific cases of a proton and a neutron targets, we have

$$\frac{d^2\sigma_p^{\nu(\bar\nu)}}{dxdy} = \frac{G^2ME}{\pi}\left\{(1-y)\frac{x}{2}\left[\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)A^0(x)\right.\right.$$

$$\left.+A^3(x)\cdot\tfrac{2}{3}(\rho^2+\lambda\rho)+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$+\frac{xy^2}{4}\left[A^0(x)\cdot\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)\right.$$

$$\left.+A^3(x)\cdot\tfrac{2}{3}(\rho^2+\lambda\rho)+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$\mp\tfrac{1}{2}x\left(y-\frac{y^2}{2}\right)[2(\tfrac{2}{3})^{1/2}(\lambda^2+\lambda\rho)S^0(x)$$

$$\left.+\tfrac{2}{3}\lambda\rho S^3(x)+\frac{2}{(3)^{1/2}}(\lambda^2+\lambda\rho)S^8(x)\right\}$$

$$\frac{d^2\sigma_n^{\nu(\bar\nu)}}{dxdy} = \frac{G^2ME}{\pi}\left\{(1-y)\frac{x}{2}\left[\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)A^0(x)\right.\right.$$

$$\left.-A^3(x)\cdot\tfrac{2}{3}(\rho^2+\lambda\rho)+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$+\frac{xy^2}{4}\left[A^0(x)\cdot\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)\right.$$

$$\left.-A^3(x)\cdot\tfrac{2}{3}(\rho^2+\lambda\rho)+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$\mp\tfrac{1}{2}x\left(y-\frac{y^2}{2}\right)[2(\tfrac{2}{3})^{1/2}(\lambda^2+\lambda\rho)S^0(x)$$

$$\left.-\tfrac{2}{3}\lambda\rho S^3(x)+\frac{2}{(3)^{1/2}}(\lambda^2+\lambda\rho)S^8(x)]\right\}$$

Averaging for the case of an isoscalar nucleon target we get

$$\frac{d^2\sigma_N^{\nu(\bar\nu)}}{dxdy}=\tfrac{1}{2}\frac{G^2ME}{\pi}\left\{(1-y)x\left[\tfrac{2}{3}(\tfrac{2}{3})^{1/2}\ (2\rho^2+3\lambda\rho+3\lambda^2)A^0(x)\right.\right.$$

$$\left.+A^8(x)\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$+\frac{xy^2}{2}\left[A^0(x)\cdot\tfrac{2}{3}(\tfrac{2}{3})^{1/2}(2\rho^2+3\lambda^2+3\lambda\rho)\right.$$

$$\left.+A^8(x)\cdot\frac{2}{3(3)^{1/2}}(3\lambda^2+3\lambda\rho+\rho^2)\right]$$

$$\left.-x\left(y-\frac{y^2}{2}\right)\left[2(\tfrac{2}{3})^{1/2}(\lambda^2+\lambda\rho)S^0(x)+\frac{2}{(3)^{1/2}}(\lambda^2+\lambda\rho)S^8(x)\right]\right\}$$

In the process of integrating this over x, we find that the axial current contribution, which is associated with the $S^k(x)$ functions, vanishes. This leads to the following result

$$\frac{d\sigma_N{}^\nu}{dy}=\frac{d\sigma_N{}^{\bar\nu}}{dy}=\frac{G^2ME}{4\pi}(y^2-2y+2)\left[\tfrac{4}{3}(\tfrac{2}{3})^{1/2}(3\lambda^2+2\rho^2+3\lambda\rho)A_2{}^0\right.$$

$$\left.+\frac{2}{3(3)^{1/2}}(\rho^2+3\lambda\rho+3\lambda^2)A_2{}^8\right]$$

where $A_2{}^0, A_2{}^8$ are constants that are related to the $A^k(x)$ functions.

This result implies that in the present light-cone model,

$$\frac{d\sigma_N{}^\nu}{dy}-\frac{d\sigma_N{}^{\bar\nu}}{dy}=0 \qquad \text{or } \sigma_N{}^\nu=\sigma_N{}^{\bar\nu}$$

This is irrespective of the explicit values of λ and ρ. We note that this equality of neutrino and antineutrino cross sections has previously been noted by Perkins (Perkins, 1974) as a general result that one expects in models containing only pure vector contributions. Experimentally one finds

$$\frac{\sigma_N{}^{\bar{\nu}}}{\sigma_N{}^{\nu}} \equiv \frac{\bar{\sigma}_0}{\sigma_0} \text{(NAL)} = 0.9 \pm 0.5$$

while

$$\frac{\bar{\sigma}_0}{\sigma_0} \text{(CERN)} = 0.46 \pm 0.12$$

These two data seem to differ greatly, and it is still not possible to conclude that $\bar{\sigma}_0$ and σ_0 are not unequal.

5. *Evaluation of* λ and ρ

Since in this light-cone model $d\sigma^{\nu}/dy$ and $d\sigma^{\bar{\nu}}/dy$ (or σ^{ν} and $\sigma^{\bar{\nu}}$) are equal, we cannot determine the numerical values of λ and ρ using the branching ratios of charged to neutral current processes

$$R = \frac{\sigma_0^{\nu N}}{\sigma_c^{\nu N}}, \qquad \bar{R} = \frac{\sigma_0^{\bar{\nu} N}}{\sigma_c^{\bar{\nu} N}}$$

However, given the values of the parameter $A_2{}^8$, we can determine from either the value of R or $\bar{R}$, the value of the ratio ρ/λ, from the following expression

$$\frac{\sigma_0 \nu N}{\sigma_c \nu N} = R = \frac{(\frac{2}{3})^{1/2}(3\lambda^2 + 2\rho^2 + 3\lambda^2)A_2{}^0 + [(\rho^2 + 3\lambda\rho + 3\lambda^2)A_2{}^8/3(3)^{1/2}]}{6[\frac{2}{3}A_2{}^0 + A_2{}^8/(3)^{1/2}]}$$

We note, however, that irrespective of the actual values of λ and ρ, the light-cone model suggests that the weak neutral current is a pure vector, which would agree with the form of the weak neutral hadronic current suggested by the Weinberg model (Weinberg 1967 and 1971).

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